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PROBABILISTIC MATCHING OF TURBOFAN ENGINE PERFORMANCE MODELS TO TEST DATA

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ABSTRACT

This paper describes the development of an improved method for reliable, repeatable, and accurate matching of engine performance models to test data. The centerpiece of this approach is a minimum variance estimator algorithm with *a priori* estimates which addresses both deterministic and probabilistic aspects of the problem. Specific probabilistic aspects include uncertainty in the measurements, prior expectations on model matching parameters, and noise in the power setting parameters. The algorithm is able to produce optimal results using any number of measurements and model matching parameters and can therefore take advantage of all measured data to produce the best possible match. This improves on current matching algorithms which require that the number of measured parameters be equal to the number of model matching parameters. This algorithm has been implemented in the Numerical Propulsion System Simulation (NPSS) and tested on a generic high-bypass turbofan model typical of those used in commercial service. The baseline engine model and simulated test data are described in detail. Several exercises are discussed to illustrate results available from this algorithm including the matching of a typical power calibration data set and matching of a typical production engine data set.

NOMENCLATURE

w_i	Prior uncertainty on independent parameters
Q	Covariance matrix on prior uncertainties
n_i	Standard deviation on measured parameters
R	Covariance matrix on measurement uncertainties
$y_{predicted}$	Model predicted value
$y_{measured}$	Measured data value
Δy	Measured-predicted residual ($y_{measured}$-$y_{predicted}$)
Δx	x-step between iterations (x_i-x_{i-1})
Δx_a	x-distance from initial starting point (x_i-x_{start})
$\Delta\hat{x}$	MVE optimal estimated solution
H	Jacobian matrix (dy_i/dx_j), derivative of match dependendent y_i with respect to match independent x_j
E{}	Expected Value Operator

(See also Fig. 3 & Table 1 for further nomenclature)

INTRODUCTION

Engine performance models are a key ingredient in engine design, development and field support processes. They are used to create and communicate performance specifications to prospective customers. They are used during the design/development process to predict operating conditions for engine components, to establish temperatures and pressures under which engine components must operate, and to support engine life calculations. They form the basis of data analysis tools used for development test and for field monitoring of engines. In short, performance models have bearing on virtually all facets of an engine's development and use.

New engine performance models usually begin as derivatives from one or more older models with some modifications applied to the components in order to account for expected differences from the "ancestor" model(s). Data to refine the models is obtained from component and/or full-scale engine tests during the development program, from a flying test bed and/or compliance flight test during the pre-certification/certification effort, and from production acceptance tests after entry into service. Some tests will feature extensive instrumentation. Others will include only the small number of sensors provided for the in-service engines.

The engine status matching process is a performance matching activity whose objective is to provide an accurate assessment of the current performance of a specific engine type [1]. It must do this without extrapolating beyond the "jurisdiction" of the data used to inform the status match. For instance, sea level data should not be used to alter expected altitude performance unless there is a sound physical basis for such an extrapolation. Such an extrapolation might easily be appropriate for some engine components while being invalid for others. The status matching process should also provide an assessment of the uncertainty in the engine's predicted performance. This understanding of uncertainty is necessary to manage risks of "over-quoting" the engine's performance or of subjecting components to harsher than expected conditions in service.

This paper describes results from a joint project between the Georgia Tech and GE Aircraft Engines whose aim is to establish a rigorous, repeatable process to update a status match based on a new set of test data. The recent work has focused on a series of exercises using a generic turbofan model. In each of these instances, simulated data was generated from the models with known, implanted faults and with simulated measurement noise added to the artificially generated data. The exercises included a series of noise-free cases to explore the response of the solution algorithm, a set of simulated production engines and a simulated development engine performance test. The emphasis of this work is to gain an understanding of the process. The issues and characteristics contained in these test cases are representative of those encountered in the fully developed cycle status-matching problem regardless of the solution technique.

Underlying Need for High-Accuracy Engine Models

Engine performance models fulfill many important roles in the design, development, marketing and support of gas turbine engines. The status matching processes in use give excellent results when measured against the data which prompted the match. In most cases, this is appropriate. In a few instances, the objective should be to provide a model that uses the current data to upgrade an existing model, while respecting the earlier results already embedded within the model. Because current practice emphasizes matching the new data, it does not rigorously assign significance to earlier data sources used in developing the existing model. Creating this "balanced" match is a difficult undertaking. To understand why this is so, consider some of the issues in the creation of a performance model.

1) Many types of data are available for generation of a performance model. This plethora of sources begins with the existing model, which was itself developed from many sources. The "upgrade" may introduce one or more deliberate design changes with supporting estimates from the design community; rig test data for one or more components; development test data which may have substantial internal sensor data; flight test data which will normally offer less complete sensor coverage but wider coverage of the flight envelope; production acceptance data characterized by high quality overall performance data but limited internal sensor data; and other, rarely used sources such as field data or overhaul acceptance data.[1]
2) Complicating these data source issues is the presence of "slave" hardware. Slave hardware includes any item in the test vehicle that will not be included in the final engine design. The modeling process must characterize this hardware so that its behavior may be separated from the final engine model. Deliberate introduction of slave hardware is less common today as companies work to shorten development/certification cycle times. The most common reason for slave hardware in today's engines is the introduction of design changes provoked by problems encountered during development.
3) Engine performance models are designed to predict engine behavior when installed on wing in the associated flight cowling. The prediction is usually idealized in the sense that inlet loss is assumed to be zero. Other simplifying assumptions may also be included.[2] Engine testing is normally performed using alternate cowling and typically with a bellmouth inlet. Thus, the test-enabling hardware represents a specific type of slave hardware that must be considered as the performance model is generated.
4) There is tremendous variation in the number and quality of sensors used for the various types of testing. Some development engines will feature large numbers of sensors at several key internal measurement stations (fan inlet and discharge, low pressure and compressor inlet and discharge and low pressure turbine discharge). Production engines will usually rely on single-element probes at these same locations, or may not have these sensors at all. Production and development testing provides measured thrust and engine airflow; neither of these is available for flight test except via indirect measurements.[3]
5) Engines may be run indoors, calling for a correction to measured scale force to compute the true gross thrust. When they are run outdoors, a "turbulence control structure" may be used to reduce the impact of ambient wind on the engine's measured performance. In either case, the thrust is not directly measured, but is inferred from the scale force using empirical adjustments.
6) Measurement errors come in various guises. The easiest to cope with is "noise" which may be hypothesized to have a mean of zero. This error can be reduced via repeat testing. Bias is a more difficult problem. There is clearly an opportunity for bias whenever single-element probes are used. However, bias can still be present even when fairly extensive samplings are available at specific measurement planes.

This list could continue ad infinitum; some other sources of variation that will not receive full treatment include engine-to-engine quality variation, changes over time of the engine design and/or manufacturing process, performance differences between part vendors, engine deterioration during testing, analysis assumptions, un-modeled, imperfectly modeled or incorrectly modeled physical phenomena, and so on.

The key point is that status matching is fundamentally a statistical estimation problem. Many types and pieces of data, each with its unique statistical characteristics, must be combined to generate a model of some, possibly evolving, physical system. A rigorous solution must acknowledge the "warts" of each individual piece of data used to develop the model. It should also provide, in advance, the basis for the next upgrade to the model; that is, an estimate of uncertainty of the newly developed model. That same estimated uncertainty may be used to quantify the risk associated with various design and marketing decisions supported by the model.

STATUS MATCHING METHODS

Current status matching methods do not treat status matching as a statistical estimation problem. Current methods are basically iterative and involve a great deal of "fine tuning" of model parameters to achieve a satisfactory match. In early models, this was purely a "cut and try" process: an engineer would manually adjust model parameters, run the model, plot predicted and measured parameters, and iterate until the match was satisfactory. It was up to the engineer's experience and

1 These are rarely used because the objective of most status matches is to predict new engine performance, not overhauled engine performance.

2 These "assumptions" (such as inlet loss) reflect the business arrangements between the engine company and the airplane manufacturer.

3 Flight test thrust and airflow are usually key measurements used in contractual agreements between the engine and airframe manufacturers. Such "measurements" typically carry greater weight in the status matching process than their accuracy justifies.

judgment to select which model parameters to adjust in order to secure a satisfactory match.

Engine performance models have become increasingly complicated over time and the accuracy demanded of these models has also increased. As a result, manual tuning of model parameters has given way to semi-automated methods. Current practice is to match performance data by using the cycle model's internal solver. These solvers use the Newton-Raphson method to vary model independents (in this case, the "tuning" parameters) until all measured-predicted residuals are driven to zero [2]. The residuals and matching parameters appear in the solver setup as a set of auxiliary dependents and independents, respectively. The user is still required to select which model parameters should be adjusted to match a given set of measured parameters. If the selected parameters are not observable based on the measurement set, the solver will be unable to invert the Jacobian matrix and therefore unable to find a solution. The only recourse in this situation is to select a different set of measurements or independents.

An additional limitation of this method is that there must be an equal number of dependent measurements and independent parameters. If redundant measurements are available, the user will be forced to "throw away" some of the measurements, at the cost of a loss in match accuracy. In short, current status matching methods are based mainly on expert experience and are labor-intensive. Furthermore, the Newton-Raphson algorithm does not yield an optimal estimate of model parameters for a given measurement set.

Although very little has been published on the subject of optimal estimators for status matching, extensive literature is available in the closely related field of gas path diagnostics. A great many optimal estimator algorithms have been proposed for this problem, most based on weighted least squares techniques [3-5]. Mathiodakis [6] gives a good overview of the basic gas path assessment problem and discusses some general techniques used in its solution. The two classes of problem are very similar, the main difference being that status matching involves new engine test data of a much greater variety than is available in the gas path diagnostics problem. It stands to reason that status matching could benefit greatly from application of the ideas developed for gas path diagnostics and this is the focus of the next section.

Probabilistic Matching Algorithm

Engine cycle models are represented mathematically as coupled systems of nonlinear equations. Provided that the initial model is a reasonable approximation of the real engine, one can typically assume that: 1) initial estimates for the unknown model parameters are reasonably close to their true values, and 2) that a smooth and continuous mapping exists between model parameters and measured outputs. When this is true, a simplified model of the engine can be obtained by linearizing the engine model about the nominal parameter settings. The basic linearized model is of the form:

$$\Delta y_i = s_i + n_i = \sum_j \frac{\partial y_i}{\partial x_j} \Delta x_j + n_i \tag{1}$$

or in matrix form:

$$\Delta y = H \Delta x + n \tag{2}$$

where s_i is the true measurement signal of interest, n_i is random measurement noise, Δx_j represents an adjustment in a model parameter (delta from base value) and Δy_i represents a single measured-minus-predicted residual to be minimized by the matching process. H is the Jacobian matrix, and relates the change in model predictions, y_i, to the change in independents, x_j. Various linear analysis methods can be used to estimate the change in model parameters required to drive the error residuals to zero (where the residual is given by $y_{i,measured} - y_{i,predicted}$). Since the model is nonlinear, a change in the model parameters forces one to successively re-linearize about the new solutions until a global minimum error is reached. As mentioned previously, current matching methods employ a Newton-Raphson solver to do this. The resulting solution is non-optimal in that it does not take advantage of all information available when estimating model parameters.

A much better estimate of model parameters can be obtained using an optimal estimator that does take advantage of the probabilistic information available in the matching process. Many algorithms have been developed for this purpose, each having unique properties. Considerable effort was expended during the course of this research effort to test a wide variety of algorithms in order to ascertain their suitability for status matching. These include weighted least squares methods [3], Kalman filtering [3], extended Kalman filtering [7], Bayesian updating [8], singular value decomposition-based methods [2,7] and others. This list is not exhaustive, as there are additional optimal estimators available. Since the intent of this paper is not to provide an overview of optimal estimators but is rather to describe how they can be applied to status matching, the interested reader should see Ref. [7] for further information on the various optimal estimator algorithms.

Of the algorithms examined, the approach ultimately selected for the engine performance matching application is a Minimum Variance Optimal Estimator (MVE) [7]. This is a well-known optimal estimator that yields an estimate of x such that the estimated solution has a minimum variance from the true (but unknowable) solution. The algorithm assumes a measurement model given by Eq. (2) and a prior model estimate given by:

$$\Delta \hat{x} = \Delta x + w \tag{3}$$

where $\Delta\hat{x}$ is the change in optimal estimate on x, Δx is the change in prior estimated x as suggested by the observed measurements, and w is the prior uncertainty on the initial estimate of x. Assuming the measurement and prior uncertainty models given in Eqns. (2) and (3), the minimum variance estimate of x is obtained by minimizing the functional:

$$J_i = \tfrac{1}{2} E\left\{ \left(\Delta\hat{x}_i - \Delta x_i\right)^2 \right\} \tag{4}$$

where J_i is the ith component of the J-functional and E{} is the expected value operator. Under these assumptions, the MVE estimator equation is given by:

$$\Delta\hat{x} = [H^T R^{-1} H + Q^{-1}]^{-1} (H^T R^{-1} \Delta y + Q^{-1} \Delta x_a) \tag{5}$$

where R is the measurement covariance matrix, Q is the prior uncertainty covariance matrix, Δy is the measured-minus-predicted (residual) vector, and Δx_a is the vector distance between the current x estimate and the starting point, given by $x_{current}$-$x_{initial}$. Note that Q is related to w as:

$$E\{ww^T\} = Q \tag{6}$$

and R is related to n as:

$$E\{nn^T\} = R \tag{7}$$

A complete and detailed development of the minimum variance optimal estimator with priors is given in Ref. [7].

Specific advantages of the MVE algorithm for status matching are: 1) relative simplicity, 2) inherent robustness with respect to singularities in the H-matrix, 3) ability to handle over- and under-determined systems, 4) use of measurement uncertainty as a normalizer on residuals, 5) ability to include estimates on prior mean and uncertainty into the solution, and 6) solution flexibility. The minimum variance optimal estimator (MVE) is not subject to restrictions on equal numbers of dependents and independents: more measurements than independents or more independents than measurements. In the former case, there are an infinite number of solutions that minimize the error residuals (the problem is underdetermined) and the MVE will use information regarding the prior means and expected deviations of the unknown model to select the solution that minimizes residuals while simultaneously having the smallest root-sum-square distance from the initial starting point. In the latter case (an overdetermined system of equations), MVE will select the solution that minimizes the root sum squared error with the smallest possible deviation from the starting point.

Impact of Operating Condition Uncertainty

An additional source of noise that must be accounted for in the matching process is uncertainty in the operating conditions. For example, measurements of ambient temperature and pressure contain measurement noise. The higher the noise level, the greater the uncertainty in the operating conditions and the less one can infer from the measurement data. Likewise, uncertainty in measured power setting parameters (such as fan speed) degrades the value of the performance measurements for inferring the true engine state. This uncertainty in operating conditions can be viewed as a degradation of the signal-to-noise ratio of the measured parameters.

The approach used here is to convert the known operating condition uncertainty into an equivalent measurement uncertainty. Operating condition uncertainty degrades the ability to infer useful information from the measurements. In other words, operating condition uncertainty effectively adds additional uncertainty to the already existing y-measurement uncertainty.

This is done by calculating an equivalent measurement uncertainty assuming that the operating condition errors are normally distributed and independent of one another. The equivalent measurement uncertainty is given by the root-sum square of the actual measurement uncertainty and the equivalent measurement uncertainty contributions from the operating conditions. For example, consider a thrust measurement with known uncertainty on ambient temperature and pressure of the test conditions. The "effective" uncertainty of the thrust measurement is given by:

$$n_{fn\_equiv}^2 = n_{meas,fn}^2 + (n_{meas,Pamb}\frac{\partial F_n}{\partial P_{amb}})^2 + (n_{meas,Tamb}\frac{\partial F_n}{\partial T_{amb}})^2 \quad (8)$$

where $n_{meas,fn}$ is the uncertainty in thrust due to load cell measurement accuracy, $n_{meas,pamb}$ is the measurement standard deviation of the measured ambient pressure, and dF_n/dP_{amb} is the sensitivity of net thrust with respect to ambient pressure, and so on. This approach generalizes to any number of operating conditions for each measurement.

Identification of Singularities

An important concern in engine status matching is the identification and treatment of singularities in the diagnostic matrix (dy_j/dx_i) of Eq. (1). Singularities arise due to the presence of unobservable parameters in the measurement-independent set. A model parameter is termed "unobservable" if the measurement set available for matching does not contain sufficient information to independently determine the optimum estimate for that parameter.

An example is estimation of HP and LP turbine loss in a status match. If inter-turbine pressure and temperature measurements are available, there will be only one combination of turbine flow and loss scalars that minimizes the measured-predicted residuals. If inter-turbine measurements are not available, it will not be possible to uniquely determine a single set of turbine loss and flow scalars that minimize residual error. Instead, the parameters are confounded.

An appealing attribute of a MVE algorithm is that it yields an optimal estimate regardless of whether some of the model parameters are unobservable for a given measurement data set. Those parameters that are unobservable are simply left alone or, if two parameters are confounded, the algorithm will move both parameters so as to minimize residuals while simultaneously minimizing the root-sum-square distance that the parameters are moved from their base point.

Implementation in NPSS

The status matching algorithm described above was implemented as a series of Numerical Propulsion System Simulation (NPSS) objects [9] as shown in Figure 1. The MatchDependent and MatchIndependent objects are analogous to the NPSS solver's own Dependent and Independent objects. The MatchDependent object is a data structure containing all information needed to define the measured/predicted parameter pairs available for matching. The MatchIndependent object is a data structure containing all information needed to define what model parameters are free to be "tuned" during the matching process. The MatchNoise object is a data structure containing information to describe uncertainty in operating conditions. The user instantiates a suitable object type for each measurement parameter, model parameter, and operating condition parameter in the matching problem. The PointMatch object is the executive program containing the MVE algorithm. It is a sequential state estimation object that matches a single operating point at a time. The PointMatch object has two primary member functions: verify() and runMatch(). The former verifies the current algorithm setup and includes a variety of error traps. The latter contains the MVE match algorithm. The algorithm calculations begin by computing equivalent measurement uncertainties for each measured parameter based on the active MatchNoise objects. It then enters an iteration loop wherein it generates derivative matrices, creates prior and measurement uncertainty covariance matrices, applies the MVE estimator equation, solves for the Δx vector, checks x-constraints and Δx limits, and executes the cycle model at the new x-point. This process repeats until convergence or until the maximum number of iterations is reached.

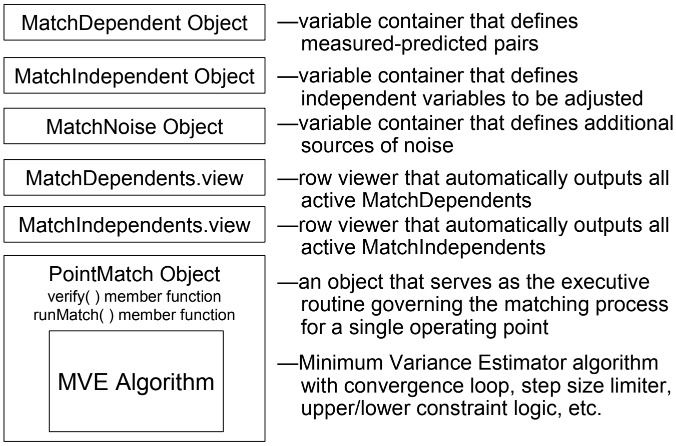

Figure 1: Structure of Matching Algorithm Implementation in NPSS.

TURBOFAN ENGINE MATCH TEST CASES

A variety of matching exercises were developed during the course of this work to test candidate status matching algorithms. The basis for these exercises was a series of simulated measurement data sets created using a generic NPSS turbofan engine model. This model is a high bypass turbofan engine typical of those used today for single-aisle commercial aircraft. The model does not match any particular engine, but is sized to be roughly equivalent to the CFM56. Engines used on two-aisle aircraft (such as CF6 or GE90) are similar in structure and are also adequately represented by these exercises.

The NPSS turbofan model uses fan speed for power management, as do all GE and CFM engines in commercial service. The major components under consideration are the fan, booster (or low pressure compressor), the high pressure compressor, the high pressure turbine and the low pressure turbine. The model is matched to the simulated measurement data by tuning scalars on loss and flow rate for each of the five components, yielding a total of ten independents.

In addition to the previous features, the generic turbofan model offers an array of parasitic flows and other features which allow generation of realistic problems for a status matching algorithm, though these are not used in the examples presented herein. The model predicts only plane average pressures and temperatures and has no modeling of sensors. These may be simulated by adding noise or bias to the plane-average quantities available from the NPSS model. The exercises included measurement suites consistent with the type of engine test being simulated (for example, a development engine test will have more and better sensors than a production engine test). Figure 2 shows the sensors assumed to be

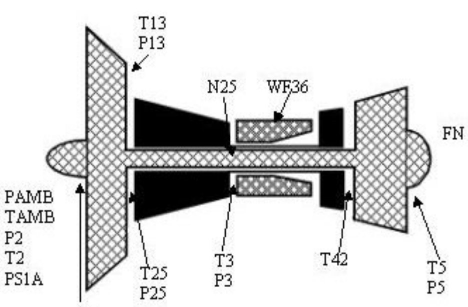

Figure 2: Location of Various Sensors Used in Test Cases.

Table 1: Assumed Engine-to-Engine Measurement Parameter Uncertainties.

	Engine Measurement	Measurement Uncertainty
1)	Ambient pressure (PAMB)[1]	0.0015 psia
2)	Ambient temperature (TAMB)[1]	0.65 deg. F
3)	Inlet total pressure (P2)[1]	0.0015 psia
4)	Inlet total temperature (T2)[1]	0.65 deg. F
5)	Fan speed (N1)[2]	0.5 RPM
6)	Inlet static pressure (PS1A)[3]	0.004 psia
7)	Core speed (N25)[4]	1 RPM
8)	HP compressor inlet temperature (T25)[5]	0.8 deg F
9)	HP compressor inlet pressure (P25)[5]	0.03 psia
10)	HP compressor discharge temperature (T3)[4]	2.5 deg F
11)	HP compressor discharge pressure (P3)[4]	0.5 psia
12)	Fuel flow (WF36)[4]	20 lbm/hr
13)	LP turbine inlet temperature, EGT (T42)[4]	10 deg F
14)	LP turbine discharge temperature (T5)[6]	2.2 deg. F
15)	LP turbine discharge pressure (P5)[6]	0.01 psia
16)	Fan discharge pressure (P13)[6]	0.01 psia
17)	Fan discharge temperature (T13)[6]	0.6 deg. F
18)	Thrust (FN)[3]	25 lbf

Notes
[1] Used to describe the test condition
[2] Power management variable
[3] Only available for sea level testing
[4] Generally available
[5] Development testing and occasionally in production
[6] Development testing only

available, with the measurement uncertainties given in Table 1.

Three of the simulated matching exercises were designed to explore behavior of the status matching algorithm in the absence of measurement error. The first contains offsets in component state variables that can be uniquely determined from error-free measurements. A second of this series varies component parameters that cannot be uniquely determined from the available sensors. For example, it is not possible to separate HP turbine efficiency, LP turbine efficiency and LP turbine flow function using the measurement set described in Table 1.[4] This exercise allows one fault at a time, or combines fully observable component faults with a single unobservable fault. The third exercise allows multiple unobservable faults for a single case. Each of these three "no-measurement-error" exercises contains a battery of test cases that includes both ground test and flight test examples along with the attendant measurement differences between the two. For example, thrust and inlet static pressure measurement data is assumed to be available for ground test data but is not available for flight test data[5].

Next, a "production engine" exercise was developed to simulate a production status match process. This consists of readings at three distinct power settings for fifty engines, for a total of 150 measurement points. The data set included engine-to-engine component variation (about a mean offset), smaller component variation between the three readings for any

4 A pressure measurement between the turbines is required to accomplish the separation. Such pressure sensors were deleted with the advent of close-coupled turbines.
5 This assumes, of course, that the altitude data is derived from an aircraft flight test. Testing in an altitude facility would allow for direct measurement of engine airflow and a semi-direct determination of engine thrust.

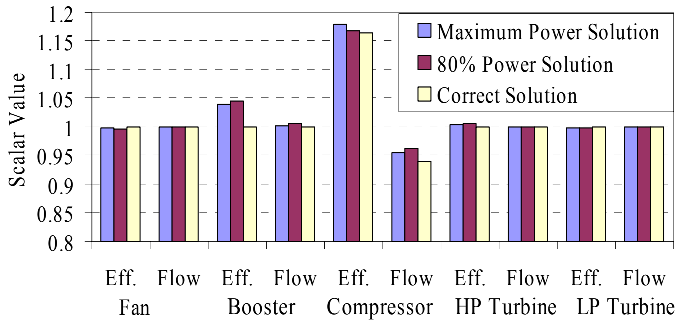

Figure 3. MVE Results for Two Power Settings and No Measurement Noise, Compressor Efficiency and Flow Fault Scenario.

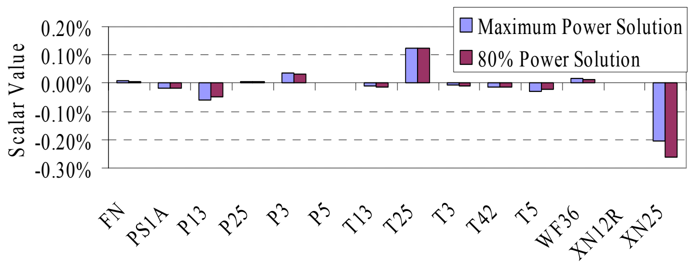

Figure 4. Error Residuals for the No-Noise Compressor Efficiency and Flow Fault Scenario.

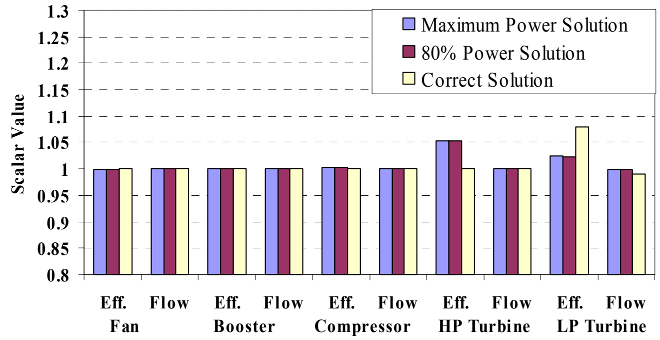

Figure 5. MVE Results for Two Power Settings and No Measurement Noise, LPT Efficiency and Flow Fault Scenario (Unobservable Faults).

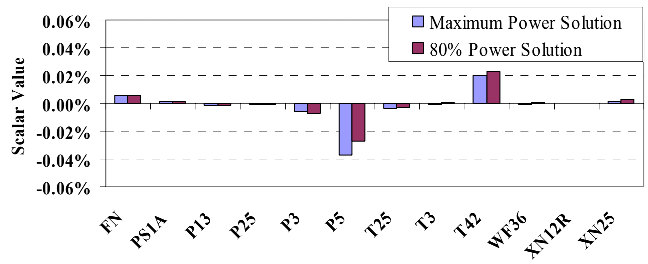

Figure 6. Error Residuals for the No-Noise LPT Efficiency and Flow Fault Scenario.

specific engine, engine-to-engine sensor bias for single-element sensors,[6] and sensor noise, including for sensors used to establish the test condition (ambient temperature, etc.). Sensor errors were chosen to be representative of the accuracy typically available in production acceptance testing.

The final exercise is a simulated engine power calibration consisting of two power sweeps for a well-instrumented engine. This exercise included all of the complicating factors described for the production engine exercise, except that no engine-to engine variation was included. Smaller sensor errors were used to reflect the improved accuracy that can be expected from development testing.

Deterministic (No-Noise) Scenarios

The status matching algorithm was first applied to a deterministic problem with no noise applied to either the match independents or dependent (measured) parameters. The purpose was to determine if the algorithm can find the solution to a problem with a known, unique answer. The simulated engine data in this exercise consisted of 14 measurements including thrust, fuel flow, core speed and various pressure and temperatures throughout the engine. Ten state variables were adjusted in the NPSS model to match the 14 measurements, yielding an over-determined scenario wherein the measurement uncertainty acts as a weighting factor on redundant measurements. Since this problem has no measurement noise, a perfect match is theoretically possible.

Typical results for this series of test cases are shown in Figure 3 and Figure 4. These figures illustrate a double-fault scenario consisting of offsets in compressor flow and efficiency. Two operating conditions are compared against the correct solution: maximum power and 80% fan speed. In this case, the optimal estimate of parameters is relatively close on both compressor loss and flow, but some of the fault has also been erroneously allocated to the booster loss and flow. The reason for this is biasing due to the prior uncertainties assumed for each of the ten independents. The prior uncertainty on component loss was assumed to be larger than the prior uncertainty on component flows by a factor of ten (this is generally a reasonable expectation for loss and flow parameters: performance is roughly ten times more sensitive to changes in flow than loss). However, this particular scenario contains a 6% flow fault in compressor flow, which is quite large relative to the prior uncertainty placed on compressor flow. Rather than allocate all of the flow fault to the compressor, the MVE algorithm distributes a portion of the fault to the booster, thereby avoiding a large fault in compressor flow while simultaneously minimizing the error residuals. This is a reasonable solution given the prior uncertainties specified in the problem definition. Note that the error between the matched cycle results and the measured data is less than 0.25% for all measured parameters, as shown in Figure 4.

Had this same scenario been run with a larger prior uncertainty on compressor flow, the resultant solution would have matched the actual fault more closely. This illustrates one of the potential pitfalls of using MVE for cycle model matching: as the prior uncertainties are increased, the MVE estimate is increasingly driven by minimization of error residuals and will tend toward the “correct” solution shown in Figure 3. Conversely, as the prior uncertainties are decreased, the algorithm is driven to minimize the deviation from the prior parameter estimates at the expense of minimization of error residuals. It is therefore imperative that the prior uncertainties be selected to accurately reflect the confidence placed in each prior estimate in order to obtain the best possible solution for the given measurements.

[6] Most of the sensors on production engines are single element probes whose primary purpose is to support the engine control system.

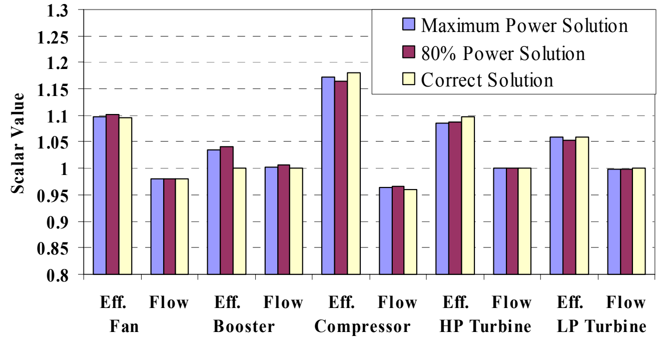

Figure 7. Parameter Estimates for Multiple Fault Scenario.

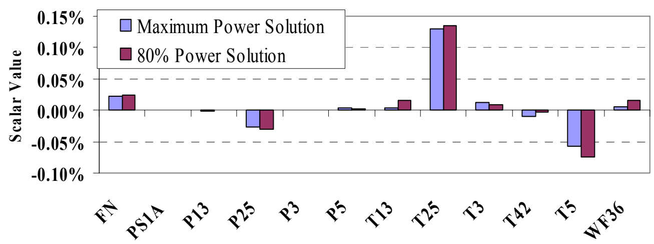

Figure 8. Error Residuals for Multiple Fault Scenario.

The second exercise set was develop to investigate MVE solutions in the presence of unobservable fault scenarios, a typical example of which is shown in Figure 5 and Figure 6 This scenario is identical to the previous except that there is a double fault in the LP turbine loss and flow scalars. In this case, HPT and LPT losses are confounded due to the measurement set and therefore the algorithm cannot find an independent solution for each. This confounding is due to the lack of an inter-turbine pressure measurement. Since the algorithm cannot independently determine the true fault for each of the losses, it does the most logical thing: it distributes the faults between both as shown in Figure 5. Even so, the overall error residuals are driven to very small values as shown in Figure 6. The measurement set for this case was not sufficient to completely resolve all ten independents, so the H-matrix contained a singularity. The standard Newton-Raphson method would not have worked for this problem, at least not without forcing the user to reduce the rank of the diagnostic matrix. The minimal variance estimator, on the other hand, has no difficulty in dealing with the singularity in the H-matrix. No special action was required on the part of the user. Overall, these results are as good as can be expected for a scenario involving unobservable parameters.

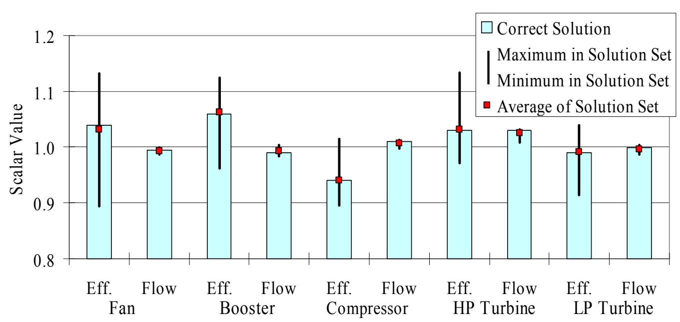

Figure 9. Power Calibration Results with Measurement Uncertainty.

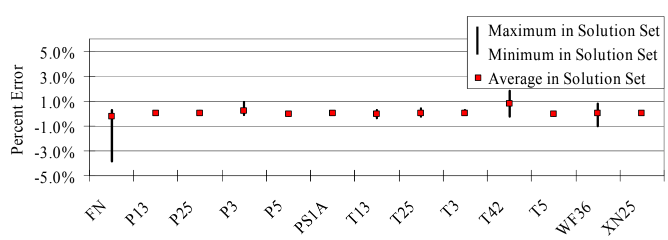

Figure 10. Error Residuals for typical power calibration results with Measurement Uncertainty.

As a final illustration of the no-noise MVE estimation results, Figure 7 and Figure 8 show typical results for a multiple fault scenario. Note that this example contains faults in every component except the booster. The MVE algorithm is able to find acceptable solutions for both the max power and part power match readings. In fact, these are in many ways better than the previous two fault scenarios in that all parameters are matched well. The largest error occurs on the booster efficiency, which is 5% higher (or about -0.5 pt efficiency) from the correct value. Once again, the measurement residuals are driven to very small values, less than 0.05% error for all but T42.

The reader should bear in mind that in any "real" matching problem, the true values of the parameters are unknowable. The "correct" solution is only knowable for these exercises because the data was synthetically created specifically for this purpose. The *only* means ordinarily available to measure the goodness of fit in a model match is how well the model predictions match the complete ensemble of measured performance data. The quality of the match shown in Figure 8 (all residuals matched within 0.05%) is quite good by any reasonable standard.

Power Calibration Results

Having established how the basic algorithm works in multiple deterministic scenarios, the next step was to examine its performance on a simulated engine power calibration. In this example, the data to be matched are from two sea level static power calibrations. A power calibration consists of an engine run at various power settings at roughly the same ambient conditions. Temperature and pressure measurements for the power calibration are derived from measurement rakes; thus, engine-to-engine bias is not included (except for the T42 measurement which is a production engine item). The exercise included simulated offsets to multiple components which were approximately (but not exactly) independent of power setting.

Typical power calibration solution results under this set of assumptions are shown in Figure 9 and Figure 10. Since this exercise is matching a series of readings ranging from low to high power, the results are presented in the form of an average solution (solid dot) and error bar representing maximum and minimum estimated solution. Figure 9 shows that the algorithm was able to find an average solution that matches the correct solution very well, even in the presence of multiple component faults. Figure 10 shows that the average error residuals between the simulated measured data and the cycle data are much less than one percent on all parameters except T42. The error on T42 is to be expected since, as was mentioned previously, T42 measurements contain measurement bias. The dispersion between minimum and maximum estimated parameters shown in Figure 9 is mainly due to low power points. Since the measurement standard deviation on all parameters was specified as an absolute value (rather than as a percent of the measured value), the signal-noise ratio at lower power settings is substantially less than for high power. The result is more variation in the estimated match parameters at low power than at high power. Nevertheless, the average match across the range of readings is accurate.

Production Engine Status Match

The final exercise conducted with the matching algorithm was a production engine match. Simulated acceptance test data consists of 50 engines, each at three readings corresponding to takeoff, maximum continuous and part-power operating conditions. Both engine-to-engine and point-to-point sensor uncertainty were included. The status match algorithm was again applied point-by-point to give an optimal estimate for each of the 150 readings, with the average over all 150 solutions being the assumed status match solution.

Typical results from the average production engine status match are shown in Figure 11 and Figure 12. This is a multiple-fault scenario consisting of fan loss and flow faults, compressor loss and flow faults, an HP turbine loss fault, and an LP turbine loss fault. Once again, the MVE solution for the production engine data matches the correct values very closely and with little error. The measurement residuals over all 150 readings are given in Figure 12 and are less than 0.2% on average. Also notice that the worst-case (minimum and minimum) residuals are never worse than 0.5% error with the exception of T42. The larger variance in T42 residual is again due to the bias inherent to this measurement.

CONCLUSIONS

The MVE-based approach to optimal nonlinear parameter estimation has significant advantages over current Newton-Raphson-based approaches used for matching of cycle models to test data. Among these are the ability to accommodate an unequal number of measurement and state parameters and the ability to account for prior knowledge of model parameters including expectation of how much they might deviate from their initial values. The MVE approach also identifies and regularizes singularities in the matching process and can therefore take maximum advantage of all the data (deterministic and probabilistic) available in the status matching problem.

The results observed from synthetic engine matching test cases are very encouraging. The MVE algorithm has the flexibility to accommodate almost any conceivable matching scenario and is capable of recovering the exact solution given an exact and observable measurement set. When given noisy data, the algorithm will find a solution that compromises between the prior parameter values and the minimum residual solution. The algorithm converges quickly and rarely gives difficulty relative to NPSS solver convergence. The algorithm was able to find a reasonable solution for all observable single-fault cases examined, found a best-compromise result for unobservable fault scenarios, and yielded very accurate match results for both the production match exercise and the power calibration exercise. In all cases, the error residuals are driven to very small values, typically less than 0.2% of their measured value.

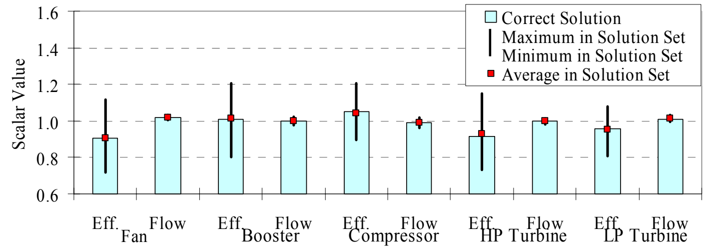

Figure 11. Match Independent Results for a Production Engine Status Match with measurement uncertainty and prior uncertainty.

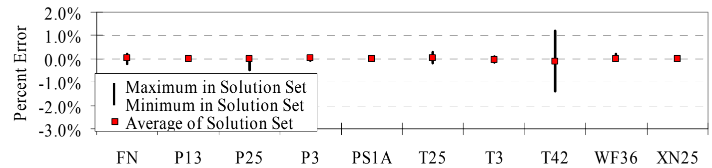

Figure 12. Error Residuals for a Production Engine Status Match with measurement uncertainty and prior uncertainty.

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